Title: Divine Ratios: A Study of the Fibonacci Sequence and Golden Ratio

Link to Outcomes:

- **Problem Solving** Students will demonstrate their ability to solve problems in mathematics, including problems with open-ended answers, problems which are solved in a cooperative atmosphere, and problems which are solved with the use of technology.

- **Communication** Students will demonstrate their ability to communicate mathematically through oral and written analysis of group test results, representing and using numbers in a variety of equivalent forms.

- **Reasoning** Students will reason mathematically by making conjectures, gathering evidence, and building arguments to support their conclusions.

- **Connections** Students will draw on their knowledge of graphs and ratios to analyze their data and draw conclusions about correlations between groups of data.

- **Geometry** Students will plot ordered pairs to represent the relationship of the length to the width in selected rectangles.

- **Measurement** Students will select the appropriate metric unit, choose the tool to find measurements, and apply those measurements to interdisciplinary problems.

- **Statistics** Students will collect, organize, display, and interpret data through bar graphs and two dimensional graphs. They will then write an evaluative paragraph about the results.

- **Arithmetic Operations** Students will write and solve simple proportions using the appropriate operation and a calculator.

- **Number Relationships** Students will apply number theory concepts, such as the Fibonacci Sequence and the Golden Ratio.

- **Patterns/ Relationships** Students will generalize a correlation between graphs of two different data sets.

Brief Overview:

This activity introduces the Fibonacci Sequence and relates the ratio of nth term to the previous term to a linear equation. They will compare the ratio of the length to the width in the selected rectangles to the ratios of n to n+1 (excluding the first three terms) found in the Fibonacci Sequence. Students then will correlate the ratio of the length of their upper body to the length of their lower body and the Golden Ratio.

Grade/Level:

Grades 6 - 8 (General Math with extensions to Social Studies and Art)

Duration/Length:

This activity should take 3 or 4 days. The activities for the second day may take longer than anticipated.

Prerequisite Knowledge:

Students need to have a basic knowledge of :

- ratio and proportions
- graphing
- metric measurement
- fractions, decimals and percents

Objectives:

- Describe and list terms of the Fibonacci Sequence
- Generate subsequent values in a sequence
- Find length using metric measurement
- Express ratios as fractions and decimals
- Convert ratios to fractions, decimals, and percent
- Compare ratios in a variety of ways
- Construct two-dimensional graphs
- Communicate hypotheses and conclusions
- Justify conjectured correlations and conclusions

Materials/Resources/Printed Materials:

- Calculator
- Pencil & Paper
- Meter Sticks
- Graph Paper
- Chart Paper for Tally Sheet
- Resource Material on Fibonacci
- Computer Software for Data Analysis (Optional)
- Index Cards

Development/Procedures:

Day 1:

- Introduce the historical figure Fibonacci.

- Give the students the first five numbers of the Fibonacci Sequence (1,1,2,3,5,...) and have them hypothesize the next three numbers (8,13,21) in the sequence.

- Have the students explain their methods for finding the next numbers.

- Share the standard Fibonacci Sequence with the class. (Any number in the Fibonacce Sequence except for the first two is the sum of its two predecessors. Ex.:. 8 + 13 = 21, 13 + 21 = 34)

- Have students list the ratio of nth term to previous term, starting with the second number in the sequence, (nth term:n-1 term) up to 233.

(1:1) = 1		(5:3) = 1.666
(2:1) = 2		(8:5) = 1.6
(3:2) = 1.5		(13:8) = 1.62

- Have the students graph ratios (<u>n-1 term:nth term</u>) up to 89 (if you have graphing calculators or computer software available, extend the graph to 233). **SPECIAL NOTE**: When you plot on a coordinate plane, the nth term **MUST** be the Y coordinate. <u>Title</u>: Fibonacci Sequence Plot.

- Assign Worksheet #1 for homework (Survey Sheet, prerequisite to following day's work).

Day 2:

- Record students' survey results from Worksheet #1 on overhead or chart paper.

- Total results from each individual rectangle. **Note**: Figures B and E have Golden Ratios. **Note**: Squares are a subset of rectangles (Figure F).

- Use data to make a bar graph comparing popularity of the various rectangles from Worksheet #1 on overhead or chart paper (Computer Software - optional).

- Discuss with students which rectangles received the greatest number of votes. Introduce the historical background on the "Golden Ratio" in architecture and art ("Golden Ratio" is 2:1 + 5 or approx. 1:1.618). Examples: The Parthenon, Great Pyramids of Ghiza, and Last Supper painting by Salvador Dali.

- Determine the ratio (shorter side:longer side) of each rectangle on Worksheet #1. Round to three decimal places (calculator is recommended).

- Plot the ordered pairs representing the ratios just determined onto the Fibonacci Sequence Plot from Day 1. Analyze for correlations between the individual ratios (rectangle) and Fibonacci Sequence ratios. **SPECIAL NOTE**: When you plot your ordered pairs, the width **MUST** be the X-coordinate.

- Assign homework to write a paragraph comparing the results of each individual's survey to the plot just completed. Then compare the class results to this plot.

Day 3:

- Share sampling of previous night's homework. Analyze results through class discussion.

- Measure body parts as indicated on Worksheet #2.

- Record A_2 and B_2 from Worksheet #2 on an index card.

- Line students up by least to greatest using the A_2 value.

- Record all A_2 and B_2 values in order of line-up on a class chart.

- Plot the ordered pairs from the chart on the Fibonacci Sequence Plot from Day 1.

- Discuss in small group any correlation that they observed. Note: A line parallel to the Fibonacci Sequence Plot may be drawn to assist with analysis.

- Write paragraph analyzing the correlation between the body ratios and the "Golden Ratio" as illustrated on the Fibonacci Sequence Plot.

- Assign homework to complete chart on bottom of Worksheet #2 and complete the paragraph from the previous step.

Evaluation:

Worksheet #1	**Total 15%**		**Criteria**
	Complete Survey	5%	(complete/incomplete)
	Expressing Ratios	5%	(1% per correct response)
	Rectangle Survey Table	5%	(complete/incomplete)
Worksheet #2	**Total 15%**		
	Measurement Activity	10%	(complete/incomplete)
	Chart	5%	(complete/incomplete)
Fibonacci Sequence Plot #1	**Total 15%** Class Activity	15%	(complete/incomplete)
Fibonacci Sequence Plot #2	**Total 15%** Individual Activity	15%	(complete/incomplete)

Written Response #1 **Total 20%**
Paragraph should include:
1) Summary of individual and class observations.
2) Observation of any correlations between expected ratio (approx. 1.618) and actual data.
3) Conclusion justified with data.

Written Response #2 **Total 20%**
Paragraph should include:
1) Summary of individual and class observations.
2) Observation of any correlations between expected ratio (approx. 1.618) and actual data.
3) Conclusion justified with data.

Extensions/Follow Up:

Students will create a Collage of Human Figures and Golden Rectangles using the following procedure. Construct and cut out four different sized golden rectangles from one 9" X 12" piece of white construction paper. Trace a duplicate of each rectangle, using a different color of construction paper for each. Cut out the colored rectangles and set aside. On one side of each white rectangle, create a textured pattern using several different colored crayons. Turn each white rectangle over to the blank side and draw a human figure which is as tall as the rectangle's length while keeping in mind the proportions and golden rectangles discovered in the initial activity. Cut out the four human figures. Assemble the human figures and rectangles in an arrangement that is "golden to their eyes" and glue individual parts on an 8" X 11" black background paper. Mount the finished design on a 9" X 12" white background paper.

Materials needed:
White Construction paper (9" X 12") for each student, black construction paper (8" X 11") for each student, at least four different colors of construction paper, pencils, rulers, scissors and glue.

Students will research a famous building from an ancient culture such as the Parthenon, the Great Pyramids of Ghiza, or the Great Temple of Karnak to find out what role the Golden Ratio plays in its architecture.
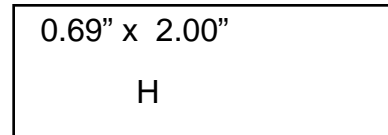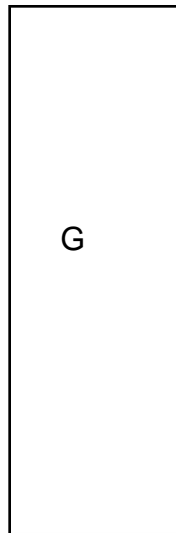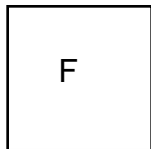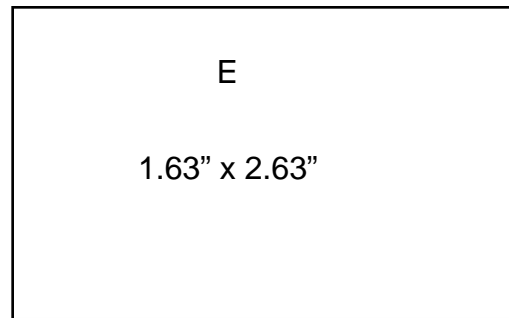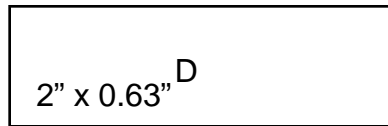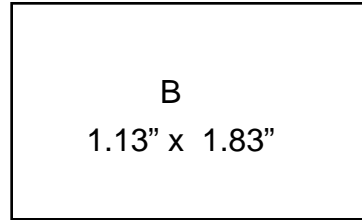
Students will perform the activity titled "Scaling Up the Human Body: Adding vs. Multiplying, Distortion vs. Proportion," from the book Equals Investigations: Growth Patterns. (ISBN 0-912511-57-5, published by Lawrence Hall of Science in 1994)

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A
1" x 1.25"

B
1.13" x 1.83"

D
2" x 0.63"

C
0.25" x 1.38"

E
1.63" x 2.63"

F
0.75" x 0.75"

G
2.75" x 0.88"

0.69" x 2.00"
H
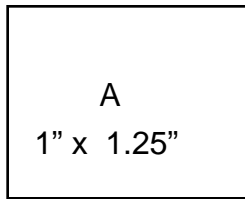
Directions: Survey five different people. Ask the participant to identify the
two rectangles that they find most pleasing to the eye. Record the
results on a separate sheet of paper.   After completing the survey,
record the total for each rectangle on the table below.

Rectangle Survey Table

Rectangle A _____   Rectangle E _____

Rectangle B _____   Rectangle F _____

Rectangle C _____   Rectangle G _____

Rectangle D _____   Rectangle H _____

Worksheet #2   Name: _____

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ARE OUR BODIES GOLDEN ?
Data Recording Sheet
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<u>Measure</u> the following <u>lengths</u> to the nearest centimeter.

Data Set I

A   = Top of head to middle of throat   _____

B   = Middle of throat to navel   _____

A_2 = A+B   = Top of head to navel   _____

Data Set II

C   = Knee cap to floor   _____

D   = Navel to knee cap   _____

B_2 = C+D   = Navel to floor   _____

Complete the chart using data from above.

	Ratio	Fraction	Decimal	Percent
A:B				
A:A_2				
B:A_2				
C:D				
C:B_2				
D:B_2				